TIME MINIMIZING SOLUTION PROCEDURES FOR THE TRANSPORTATION PROBLEM WITH MIXED CONSTRAINTS

A Synopsis

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by

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1. Introduction

Distribution of goods produced by a factory from various warehouses to different markets where it is required is a common problem, which is referred as transportation problem. It is a special class of linear programming, which is related with day-to-day activities in our real life. Transportation problem deals with the problems on distribution and transportation of resources from one place to another. The goods are transported from a set of sources (i.e. factories) to a set of destinations (i.e. warehouses) to meet the requirements. Basically, the purpose is to minimize the overall cost of transporting goods from one place to another satisfying demand and supply requirements.

The classical transportation problem is one of the many well-structured problems in operations research which is defined as a transportation problem with equality constraints in which each source is restricted to supply fixed amount of goods to meet the fixed requirement of respective destination. Whenever the total availability of supply an amount of commodity is equal to the total demand to be fulfilled, then the problem is classified as balanced transportation problem. But if the total availability of a commodity from each source is not equal to the total demand that required by the destination, then the problem comes under the unbalanced transportation problem. So as to make the problem balanced, a dummy supply or dummy demand is inserted with the associated cost zero.

Practically, the problems having such restrictions will not be always possible. Different sources and destinations impose different restrictions on supply and demand respectively according to their availability and requirement. Some sources want to supply exactly the fixed amount of goods. But some other sources do not want to supply the amount less than a fixed amount. This case arises for the big factories, companies etc. Rest other sources are not able to supply more than a fixed amount of goods. This case arises for small factories, companies etc. Similar conditions occur for the destinations also. Due to the sources and destinations having mixed nature, the problem is said to be a transportation problem with mixed constraints. In real life, most of the problems are related to mixed constraints accommodating many applications as in industry, planning, communication network, scheduling, transportation and allotment etc.

The problem of minimizing the total cost of transportation has been studied since long and is well known but in modern age, people feel that time is more important than money. Money lost can be recovered but not the time lost. As Anon said, “One thing you

can't recycle is wasted time". So, it is also important that the time required for transportation should be minimized.

The time minimizing transportation problem is to find the amount of a uniform (homogeneous) commodity transported from a source (i.e. factory where the commodity is manufactured) to the respective destination (i.e. warehouse where the commodity is to be distributed) satisfying the given supply and demand limits so that the transportation work is to be finished in minimum time. This problem actually minimizes the maximum of transportation time needed between a supply point and a demand point such that the distribution between the two points is positive. The time minimizing transportation problem has a great importance in such cases-

(i) In military transportation, where at the time of emergency, the time of transportation is of prime consideration.

(ii) In the transportation of perishable goods such as fresh fruits, vegetables etc., where a delay in transportation may result in much larger loss than any cost advantage attained by transporting at lowest cost.

The aim is to minimize the time of transportation which remains independent of the amount of commodity transported. It is assumed that

(i) the carriers have sufficient capacity to carry goods from an origin to a destination in a single trip and,

(ii) they start simultaneously from their respective origins.

For the time minimizing transportation problem, the transportation of a homogeneous product from source to destination is done in parallel and its prime aim is to supply the required quantity of product to the destination within the shortest time. Thus, the basic difference between cost minimizing and time minimizing transportation problem is that the cost of transportation changes with variations in the quantity but the time involved remains unchanged irrespective of the quantities transported.

Mathematical Formulation

Transportation problem can be seen mathematically by introducing the following terminologies:

$O_i = \text{i}^{\text{th}}$ supply point , $i = 1,2.....m$

D_j = j[th] demand point , $j = 1,2.....n$

a_i = amount of commodity available at supply point O_i

b_j = amount of commodity required at demand point D_j

c_{ij} = transporting cost per unit commodity from supply point O_i to demand point D_j

x_{ij} = amount of commodity transporting from supply point O_i to demand point D_j

m = number of supply points

n = number of demand points

t_{ij} = time required for transporting x_{ij} amount of commodity from supply point O_i to demand point D_j

The mathematical model for transportation problem with mixed constraints is to find x_{ij} which

$$\text{Minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Subject to the constraints

$$\begin{cases} \sum_{j=1}^{n} x_{ij} \geq a_i, & i \in \alpha_1 \\ \sum_{j=1}^{n} x_{ij} = a_i, & i \in \alpha_2 \\ \sum_{j=1}^{n} x_{ij} \leq a_i, & i \in \alpha_3 \\ \sum_{i=1}^{m} x_{ij} \geq b_j, & j \in \beta_1 \\ \sum_{i=1}^{m} x_{ij} = b_j, & j \in \beta_2 \\ \sum_{i=1}^{m} x_{ij} \leq b_j, & j \in \beta_3 \\ x_{ij} \geq 0 \end{cases}$$

where, $\alpha_i > 0, \forall i \in I$; $\beta_j > 0, \forall j \in J$

$I = \alpha_1 \cup \alpha_2 \cup \alpha_3 = \{1,2.....m\} =$ set of index of supply points.

$J = \beta_1 \cup \beta_2 \cup \beta_3 = \{1,2.....n\} =$ set of index of demand points.

The mathematical model for time minimizing transportation problem is to find t_{ij} and x_{ij} which

(P) Minimize Z = $\max_{i,j} \{t_{ij}; x_{ij} > 0\}$

Subject to the constraints

$$\begin{cases} \sum_{j=1}^{n} x_{ij} \geq a_i, & i \in \alpha_1 \\ \sum_{j=1}^{n} x_{ij} = a_i, & i \in \alpha_2 \\ \sum_{j=1}^{n} x_{ij} \leq a_i, & i \in \alpha_3 \\ \sum_{i=1}^{m} x_{ij} \geq b_j, & j \in \beta_1 \\ \sum_{i=1}^{m} x_{ij} = b_j, & j \in \beta_2 \\ \sum_{i=1}^{m} x_{ij} \leq b_j, & j \in \beta_3 \\ x_{ij} \geq 0 \end{cases}$$

where, $\alpha_i > 0, \forall i \in I$; $\beta_j > 0, \forall j \in J$

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2. Survey of Published Research

In this section, the study carried out so far on the transportation related problems has been presented, as reviewed from published research.

The transportation problem was first formulated mathematically by F.L. Hitchcock in 1941, and discussed in detail by the Nobel Laureate T.C. Koopmans in 1947 paper. The linear programming formulation of the transportation problem and associated solution procedure were first given by G.B. Dantzig in 1951.

Brigden (1974) [5] considered the transportation problem with mixed constraints. He solved this problem by considering a related transportation problem having two additional supply points and two additional destinations. Isermann and Bielefeld (1982) [7] also studied the transportation problem with mixed constraints. They developed the set of transformation rules to convert the problem into standard transportation problem in which availability or requirement constraints are specified as inequalities can be solved by means of related standard transportation problems. To each transportation problem with mixed constraints, a standard transportation problem with two additional constraints can be related. In 2006, Veena et al. [1] had developed a heuristic method for solving transportation problems with mixed constraints which is based on the theory of shadow price. The solution obtained by the heuristic method introduced by Veena et al. is an initial solution for the transportation problem with mixed constraints.

After that Pandian and Natarajan [16] (2010) developed a new method for finding an optimal more-for-less (MFL) solution for transportation problem with mixed constraints by zero point and modification method. Note that more-for-less solution means more supply on less transporting cost. Another researcher Mondal et al. (2012) [12] also developed a method to find an optimal more for less (MFL) solution for the transportation problem with mixed constraints.

ASM-Method, a new method for finding an optimal solution for a wide range of transportation problem directly was developed by Quddoos et al. [21] (2012).

Sudhakar et al. (2012) [25] also developed a different approach namely zero-suffix method for finding an optimal solution for transportation problems with equality constraints directly. This method avoids the degeneracy problem.

Williams [26] developed a method in 1986, called the Fourier elimination method for solving the linear programming problems based on the Fourier elimination technique which generates more additional and redundant constraints. Kanniappan and Thangavel (1998) [9] modified the Fourier elimination method developed by Williams for solving linear programming problems by choosing a variable for elimination, which generates minimum number of constraints in each step compared to that of the method of Fourier elimination method. Later on in 2010, Pandian and Natarajan [19] developed a method for finding an optimal solution of transportation problem with mixed constraints using Kanniappan and Thangavel approach.

Klingman and Russell (1975) [10] had given a generalization of standard transportation model in which the origin and destination constraints were of mixed type. They obtain a model equivalent to a standard transportation problem having only one additional origin and destination.

A new method for finding an optimal solution of fully interval integer transportation problem is developed by Pandian and Natarajan (2010) [17]. They used the separation method for solving transportation problem.

In 2011, Pandian and Anuradha [14] invented a new method namely, floating point method for finding an optimal solution to a transportation problem with additional constraints. This method has an advantage that the optimal solution is not necessarily to have (m+n-1) allotted entries.

Alam and Rastogi [2] (2011) presented the need of transportation problems, the historical use and developments in the area of transportation problem.

Bhatia et al. [4] (1976) developed the technique for minimizing time in a transportation problem with equality constraints. The procedure involves finite number of iterations and is based on moving from an initial basic feasible solution to another till the last solution is arrived. Then in 1993, Mathur et al. [11] focused on the bi-criteria Bottleneck linear programming problem. It has been established that all the efficient pairs of values of the objective function can be attained at the extreme points of the feasible region.

Pandian and Natarajan (2011) [20] also proposed the new method i.e. blocking method for finding an optimal solution to Bottleneck transportation problems and one another method namely, blocking zero point method which is based on zero point method for cost-time minimizing transportation problem with equality constraints. M. Sharif Uddin (2012) [22] also developed an algorithm to determine the minimum transportation time. In this algorithm he determined the initial basic feasible solution using distribution indicators.

Arora and Puri (1997) [3] presented the lexicographic versions of the cost minimizing transportation problem (CMTP) and time minimizing transportation problem (TMTP). In this paper, a standard cost minimizing transportation problem is formulated for finding lexicographic optimal solution of CMTP and TMTP.

Sonia and Puri (2004) [24] considered the two level hierarchical balanced time-minimizing transportation problems which is a concave minimization problem (CMP). A polynomial bound algorithm is developed in this paper.

The total transportation time problem regarding the time of the active transportation routes was studied by Ilija Nikolic [13] in 2007. He developed methods of generation of the optimal solution in selected cases.

Sharma et al. (2010) [23] considered the capacitated two-stage time minimization transportation problem in which the total availability of a homogeneous product at various sources more than the total requirement of the same at destinations.

Pandian and Anuradha (2011) [15] developed the split and bound method for solving the fully interval transportation problem with additional impurity constraints which is based on floating point method. The method has been developed without considering the mid-point and width of the intervals.

Fegade et al. (2011) [6] developed a new method for finding an optimal solution for transportation problem using interval and triangular membership functions.

The time minimizing transportation problem with fractional Bottleneck objective function (TMTP-FBOF) was presented by Jain and Saksena [8] in 2012. They developed an algorithm to determine an initial efficient basic solution for TMTP-FBOF.

3. Proposed Work

We have gone through a number of research papers on transportation related problems. Out of them, some of the recent works cited below are the source of inspiration to do the research work. In recent times, the problem of minimizing the duration of transportation has been dealt by some researchers in transportation problem with equality constraints. Thus, we come to the conclusion that the problem of minimizing the duration of transportation in the transportation problem with mixed constraints has not yet been dealt and this itself is the main motivating force for the present study.

We propose to develop proper and efficient methods for finding an optimal solution of time minimizing transportation problem (P) as given in introduction section.

In order to meet our objective blocking method will be used, as pointed out by Pandian and Natarajan (2011) [20] for finding an optimal solution to bottleneck or time minimizing transportation problem with equality constraints. In this method, they block some cells which have more time unit than the maximum of row and column minima from time minimizing transportation table and then allot the maximum possible to a cell having only one unblocked cell in the row/column. And then, modify the active transportation table and repeat the process until the required optimal solution is obtained. We are going to extend this idea for minimizing the duration of transportation in time minimizing transportation problem with mixed constraints.

As indicated by Bhatia et al. [4] the time can also be minimized in a transportation problem with equality constraints by the movement of cell. In their approach, firstly they found the initial basic feasible solution which can be obtained by any existing method used for cost minimizing transportation problem. Then, search for the basic cell having greatest time and try to exchange that basic cell with a non-basic cell having time smaller than that basic cell to get the required optimal solution in finite number of iterations. This approach will also be used with time minimizing transportation problem with mixed constraints. By applying any suitable method of cost minimizing transportation problem with mixed constraints we will search basic feasible solution and then the method of improving the solution will be developed.

Other approach is to assign firstly the maximum amount to the cell having least time according to the demand and supply limits. Then, to the cell having the next least

time satisfying the demand and supply limits. In this way, we can obtain a basic feasible solution to the problem but not optimal. Then, locate the maximum time corresponding to the basic cell and try to make it non-basic by shifting the amount supplied to it to the other cell having time lower than that. This can be done through by making loops or elsewhere. In this way, we need to detect that stage from where it is not possible to exchange a basic cell with a non-basic cell to reduce the time. That is the optimality condition will be developed.

As Pandian and Natarajan (2010) [18] proposed the method for finding an optimal more-for-less solution for transportation problem with mixed constraints by changing all inequalities in to equalities with the lowest possible feasible supply and demand limits. Then, by creating the Modi index they checked the optimality of the solution. It is hoped that using this idea, the duration of transporting commodity in time minimizing transportation problem with mixed constraints will also be minimized.

Other approaches may also be adopted further to minimize the duration of time in time minimizing transportation problem.

Algorithms will be developed for finding the optimal solution of time minimizing transportation problem with mixed constraints by the methodologies discussed above. Various numerical illustrations will be carried out for testing the efficiency of the algorithms.

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